Physics and chemistry

Entry Exam 0

Length: 3 hours

Centre International de Valbonne Entry Exam 0

Name and surname: ____

School's name: ____

Instructions to candidates

- Write your name and school affiliation above.
- Do not open the examination paper until instructed to do so.
- For each answer indicate the full question number.
- A value without the good unit is considered as false.
- Number the pages.
- Non-programmable calculators are allowed.
- During the exam, if you find a mistake in the subject, write it on your paper and explain the changes needed to go on.

Ice cube physics

I. Movement of the ice cube inside the glass

The ice has a cubic shape with a edge length **a = 2.0 cm** and a mass **m = 7.36 g**.

I.1.a. What is the density $\boldsymbol{\rho_0}$ of liquid water at usual temperature and pressure into SI units?

I.1.b. What is the density $\boldsymbol{\rho}$ of the ice cube? Compare with that of liquid water. Comment on the result.

The ice cube is dropped into a glass filled with a height **h = 10.0 cm** of liquid water. The cube goes down and reaches the bottom before going back to the surface. We use a Cartesian coordinate system with the (Oz) vertical axis pointing down. We choose the origin O on the surface of the water. We consider **z the depth of the center of mass** G of the ice cube (Figure 1.). **In part I, the melting of the ice is not considered.**

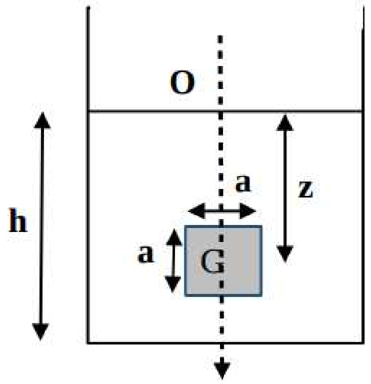


Figure 1

We first study the buoyancy of the ice cube when totally under the surface of the liquid water.

I.2.a. What is the formula of the buoyancy force using the previous Cartesian coordinate system as a function of the density $\boldsymbol{\rho_0}$ of the liquid, the edge length **a** of the cube, and the gravity **g** ? What is the origin of the buoyancy force?

I.2.b. We consider the layer of water placed between $z+\frac{a}{2}$ and $z-\frac{a}{2}$. What are the three forces applied on it ? Draw a scheme of the situation.

I.2.c. We consider the layer of water is in equilibrium. Using Newton's second law, what is the relationship between the pressure on the top of the layer $P\left(z-\frac{a}{2}\right)$, the pressure on the bottom of the layer $P\left(z+\frac{a}{2}\right)$, the edge length of the cube **a**, the density of the liquid $\boldsymbol{\rho_0}$ and gravity **g**?

I.2.d. What is the resultant pressure force acting on the ice cube? Compare to the buoyancy force and conclude.

We now investigate the ice cube moving up from the bottom of the glass.

I.3.a. What is the relationship between **z, h and a** when the cube touch the bottom of the glass (initial time)? Same question when the top of the ice cube reaches the surface of water (final time).

I.3.b. What are the properties of the different forces acting on the ice cube? We don't consider the friction forces.

I.3.c. What is the formula of the potential energy linked to each force? For the buoyancy force, you can proceed like with the weight force.

I.3.d. Using the mechanic energy law, what is the relationship between the velocity $\mathbf{v_0}$ of the ice cube when its top reaches the surface of water and the other parameters **a,** $\boldsymbol{\rho_0}$**, h and g** ?

I.3.e. How does this velocity $\mathbf{v_0}$ evolve if there are air bubbles in the ice cube? Same question if we consider the friction forces ?

In the following, the time when the top of the ice cube reaches the surface is taken as the new initial time and velocity $\mathbf{v_0}$ as the new initial velocity.

The ice cube starts to oscillate vertically on the surface of water. One part of the ice cube is always outside water (Figure 2.). First, we will not consider friction forces.

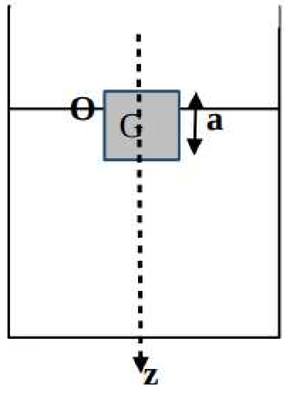


Figure 2.

I.4.a. What is the formula of the buoyancy force using **z, a, g and** $\boldsymbol{\rho_0}$ in this case?

I.4.b. Using Newton's second law, what is the relationship between **z** and $\frac{d^2z}{dt^2}$?

I.4.c. We consider a sinusoidal movement. The position **z** as a function of the time **t** is given by $z(t) = Acos(\omega t + \varphi)$. What is the name of **A,** $\boldsymbol{\omega}$ **and** $\boldsymbol{\varphi}$ in this expression?

I.4.d. Using the **I.4.b. equation**, write omega ω as a function of $\boldsymbol{\rho_0}$**, m, a and g.** Using this formula, what is the expression of period **T** ? What is the value of T?

I.4.e. What is the expression of **A and φ** as a function of $\mathbf{v_0}$**, a and ω?** What is the value of A?

I.4.f. Draw the graph of height **z** against time **t** and give the scale.

We consider now fluid friction given by $\vec{f} = -\alpha\vec{v}$ where α is supposed to be known and $\vec{v}$ is the velocity of the ice cube.

I.5.a. What is the new relationship between z and $\frac{d^2z}{dt^2}$?

I.5.b. We consider that position z as a function of time is given by $z(t) = Be^{-t/\tau}cos(\Omega t + \psi)$. What is the relationship between **τ and m, a,** $\boldsymbol{\rho_0}$ **and g** ? Same question for **Ω and m, a,** $\boldsymbol{\rho_0}$ **and g**.

I.5.c. Draw the graph of position **z** against time **t** and be careful with the initial conditions.

II. Electric circuit analogy

By using **Z = z – z$_{eq}$** where z_{eq} is the equilibrium position of the ice cube, we can obtain the equation below:

$$\frac{d^2Z}{dt^2} + \frac{\omega_o}{Q}\frac{dZ}{dt} + \omega_o^2 Z = 0(1)$$

where $\frac{\omega_o}{Q} = \frac{\alpha}{m}$ and $\omega_o^2 = \frac{\rho_0 a^2 g}{m}$

One can make the analogy between mechanical systems and electrical systems. In this part, we will study the electric circuit that is analog to the ice cube oscillating on the surface of liquid water.
We consider the two-terminal electrical components MN made of a resistor (value R), a coil (inductance value L and resistor value r considered as negligible), and a capacitor (value C). The resistor value of the cables is considered as negligible.

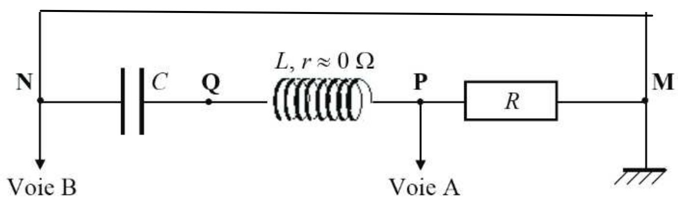


Figure 3. RLC series circuit

II.1. Energy balance in electronics

II.1.1. What is the expression of the energy stored in the inductor **L**as a function of the current **I**? In what form is this energy stored?

II.1.2. What is the expression of the energy stored in the capacitor **C** as a function of the electric charge **q**? In what form is this energy stored?

II.1.3. What is the expression of the power dissipated in a resistor **R** as a function of the current **I** ? In what form is this energy dissipated? What is the name of this phenomenon?

II.2. Energy balance in mechanics

II.2.1. What is the expression of the **kinetic energy** of the ice cube as a function of the mass **m,** and velocity **v**?

II.2.2. Explain why the expression of the buoyancy force is **F = - k z**. What is the expression of **k** as a function of **a, g and** ρ_0?

II.2.3. What is the power dissipated by the friction forces as a function of α and the velocity **v** of the ice cube?

II.3. Analogies

II.3.1. Using the previous questions, explain why the electrical circuit of Figure 3 is a good analogy with the mechanical oscillator studied in the first part. Identify the mechanical analog of each part of the circuit.

III. Using ice cube to cool down water

The ice cube is now stabilized, and begins to melt. We will only consider energy exchange between the ice cube and liquid water (we suppose the system {ice cube + water} in a thermally insulated container). The initial temperature is $\mathbf{T_i = 25°C}$ for the liquid and $\mathbf{T_0 = 0°C}$ for the ice cube. The latent heat of fusion of the ice is $\mathbf{L_f = 334\ kJ.kg^{-1}}$ and the specific heat capacity of liquid water is $\mathbf{c_e = 4.18\ kJ.kg^{-1}.K^{-1}}$. The volume of liquid is **V = 500 mL**. The density of liquid water is $\boldsymbol{\rho_0}$ and the mass of the ice cube is **m = 7.36 g.**

First, we will only consider the melting of the ice.

III.1.a. Why does the temperature of the ice remain constant during melting? Give an example of phase change that occurs at non-constant temperature.

III.1.b. We consider the ice cube totally melt. What is the expression of the **energy received** by the ice cube as a function of **m and $\mathbf{L_f}$** ?

We now consider the evolution of temperature of the liquid. We will study it as two systems:

- Liquid water "A" comes from the total melting of the ice. Initial temperature T_0
- Liquid water "B" comes from the liquid to cool. Initial temperature T_i

III.2.a. Explain why these two systems have the same final temperature $\mathbf{T_f}$**.**

III.2.b. What is the relationship between the variation in energy of the liquid water "A" ΔE_A and the different parameters **$\mathbf{T_0}$, $\mathbf{T_f}$, m and $\mathbf{c_e}$**. Without calculus, could you know if this value is positive or negative?

III.2.c. What is the relationship between the variation in energy of the liquid water "B" ΔE_B and the different parameters **$\mathbf{T_i}$, $\mathbf{T_f}$, V,** ρ_0 **and $\mathbf{c_e}$**. Without calculus, could you know if this value is positive or negative?

III.2.d. Using the results of questions **III.1.b, III.2.b, III.2.c**, what is the expression of the final temperature $\mathbf{T_f}$ of the whole system?

III.2.e. Calculate the value and comment on the result. Is the hypothesis "ice cube totally melt" correct? Explain which important phenomenon is not considered in this model. Would the real final temperature be higher or lower?

III.3. What happens to the level of water as the ice melts? Does it rise, stay the same or lower? Explain precisely using the method of your choice.

IV. Chemistry of the ice

Ice is made of water. The quality of water depends on three parameters: pH, alkalinity, and hardness.

To respect standards and especially drinking water standards, we have to measure the concentration of many chemical species such as hydrogen carbonate ion.

The goal of this part is to know if water coming from the melting of an ice cube respects drinking water standards.

- **Alkalimetric title (TA) and full alkalimetric title (TAC)**

In drinking water, alkalinity is mainly due to carbonate ions CO_3^{2-} and hydrogen carbonate ions HCO_3^-.

Alkalinity is measured by titration with a strong acid. By convention, we measure alkalinity by measuring the alkalimetric title (TA) and the full alkalimetric title (TAC) both in French hardness (°f).

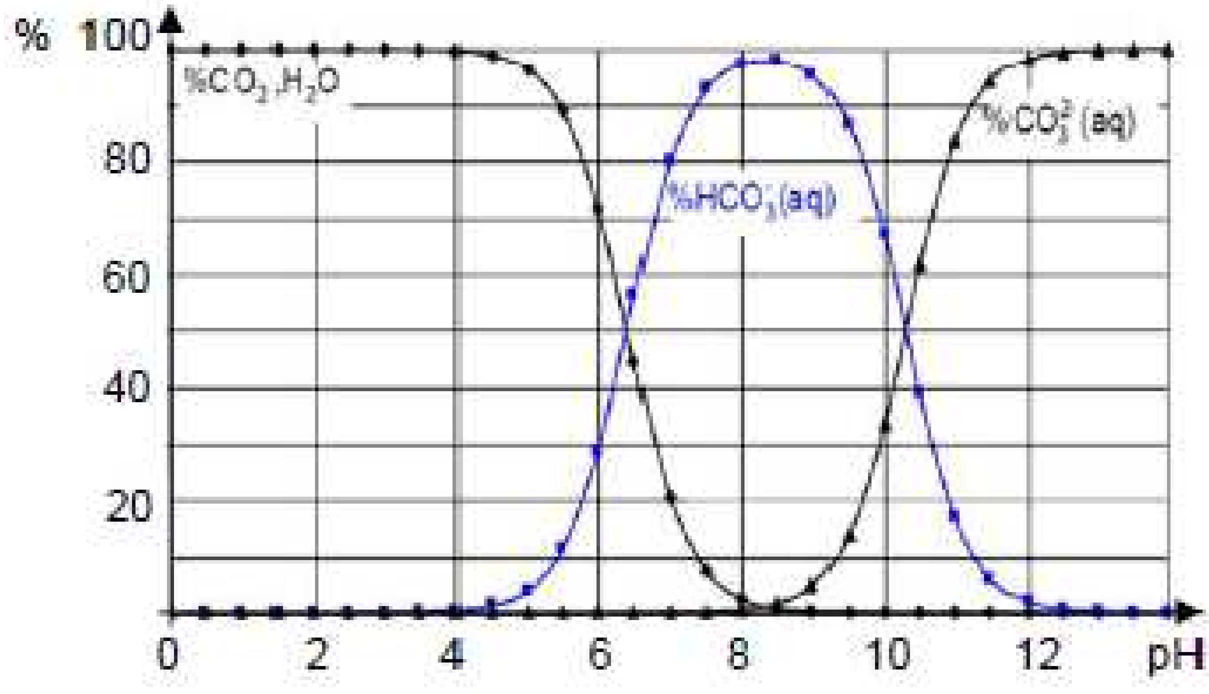
The alkalimetric title of a water sample measures the concentration in carbonate ions and in strong bases by titration using phenolphtalein as an indicator.

The full alkalimetric title of a water sample measures the concentration in carbonate ions, in strong bases and in hydrogen carbonate ions by titration using bromocresol green as an indicator.

The full alkalimetric title (TAC) in French hardness (°f) is the volume (in mL) of strong acid at a concentration C_A = 0.0200 mol.L^{-1} in oxonium ions (H_3O^+) needed to reach the endpointby titration of 100.0mL of water using bromocresol green as an indicator. The TAC is measured easily if the pH is lower than 8.2 because in this case, water contains almost exclusively hydrogen carbonate as a base and contains practically no carbonate ions CO_3^{2-} . For a sample like this, a TAC value of 1 french hardness (°f) means 12.2 mg.L^{-1} in hydrogen carbonate ions HCO_3^-. For drinking water, the TAC must be lower than 50°f

- **Molecular weight of hydrogen carbonate ion HCO_3^- :** M = 61.0 g.mol^{-1}

- **Distribution diagram of the different carbonic species**(aqueous CO_2 known as CO_2,H_2O or H_2CO_3, HCO_3^- and CO_3^{2-}) against pH.

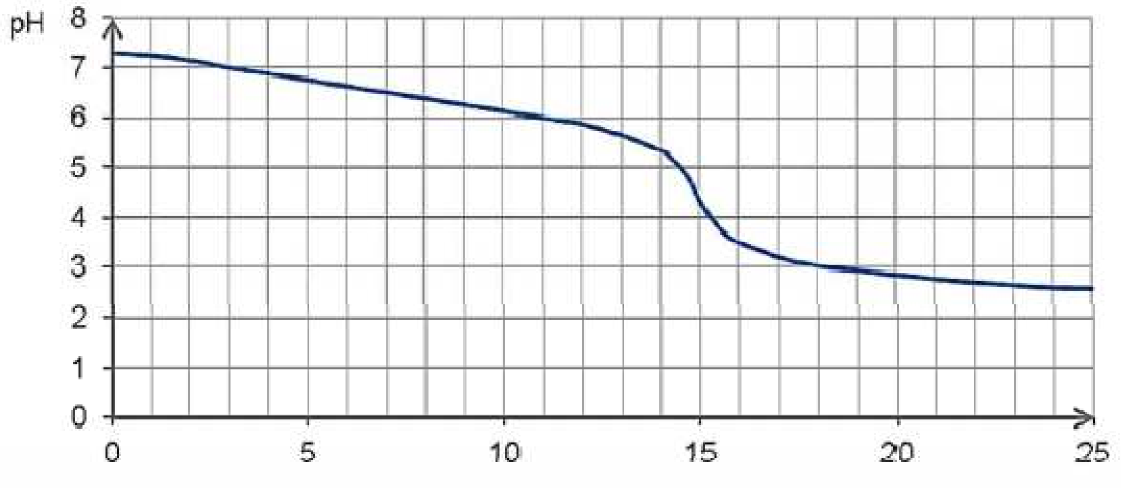


- **Acid/Base indicators**

Acid/Base indicator	Color		pH range
	Acid form	Basic state	
Bromophenol blue	Yellow	Blue	3.1 – 4.5
Bromocresol green	Yellow	Blue	3.8 – 5.4
Bromothymol blue	Yellow	Blue	6.0 – 7,6
phenolphtalein	Colorless	Pink	8.2 – 10.0

- **Water sample analysis realized by a lab technician**

Water sample volume: V = 50.0 mL
Strong acid solution: hydrochloric acid at a concentration $C_A = 0.0200$ mol.L^{-1} in H_3O^+.
The titration is realized by pH metry. The curve below shows the pH against the added volume of strong acid pH = f(V_A) during this experiment.



IV.1. Write the chemical equation for each of the two possible reactions of hydrogen carbonate with water.

IV.2. Using the distribution diagram, determine the two pKa of the carbonic species. On a pH axis, show the predominent carbonic specy for each pH domain.

IV.3. Explain the following sentence using the documents:
The TAC is measured easily if the pH is lower than 8.2 because in this case, water contains almost exclusively hydrogen carbonate as a base.

IV.4. Write the equation of the reaction made by the technician. Explain why this reaction is quantitative.

IV.5. Draw the scheme of the titration setup. Precise the location of the different chemicals.

IV.6. Explain why the bromocresol green is used as pH indicator for this titration.

IV.7. What kind of glassware must be used to take the volume **V = 50 mL**?

IV.8. Using the titration curve, what is the volume of hydro chloric acid consumed when the endpoint is reached ?

IV.9. Calculate the molar concentration and then the massic concentration of hydrogen carbonate ion in the sample.

IV.10. Using the previous answer, calculate the TAC of this water.

IV.11. Does the water analyzed here respect the drinking water standards ?